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# TAMING THE DISTANCE CONJECTURE

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*based on arXiv:* ***2206.00697***
*with* ***Thomas Grimm, Chongchuo Li***

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# THE SWAMPLAND DISTANCE CONJECTURE

[Ooguri, Vafa, 2006]

Consider an EFT, valid up to the cutoff $\Lambda_{\mathrm{EFT}}$, endowed with a set of moduli $\varphi^i$ and described by the action:

$$S^{(D)} = M_{\mathrm{P}}^{D-2} \int \mathrm{d}^D x \sqrt{-g} \Big( \frac{1}{2} R - \frac{1}{2} G_{ij}(\varphi) \partial_\mu \varphi^i \partial^\mu \varphi^j + \dots \Big)$$

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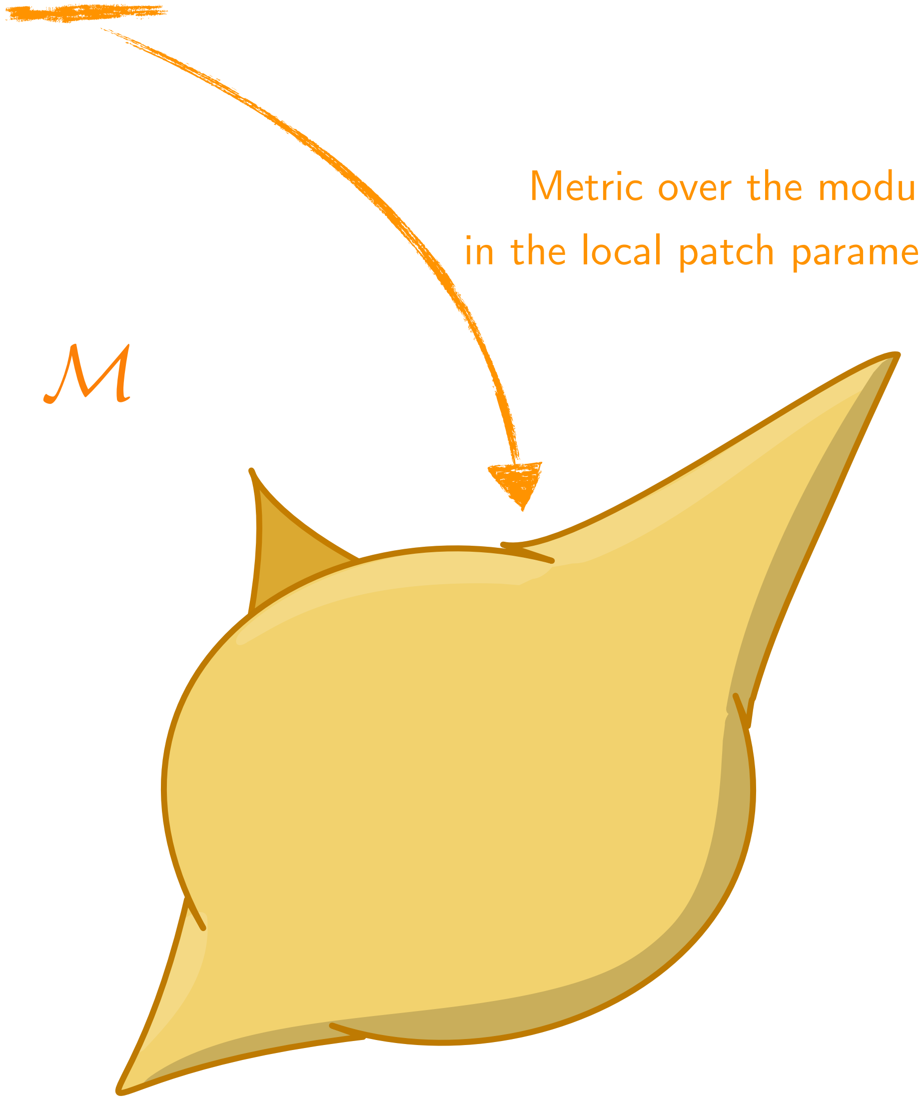

Metric over the moduli space,
in the local patch parametrised by $\varphi^i$

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## SWAMPLAND DISTANCE CONJECTURE

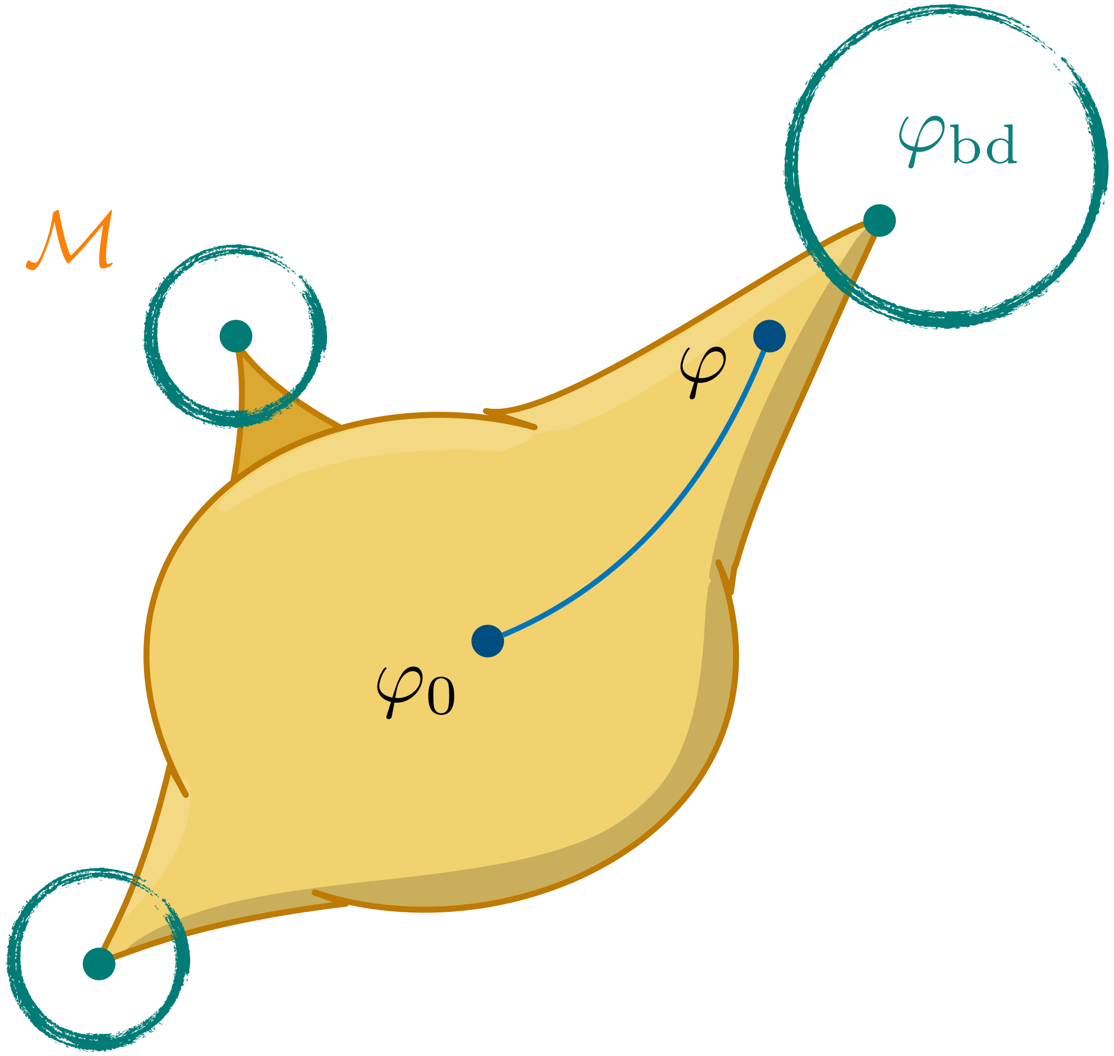
***1.*** The geodesic distance is not upper bounded:

$$\forall C > 0, \varphi_0 \in \mathcal{M} \qquad \exists \varphi \in \mathcal{M}$$

such that

$$d(\varphi_0, \varphi) > C$$

**⇒ There exist some boundaries located at infinite field distance.**



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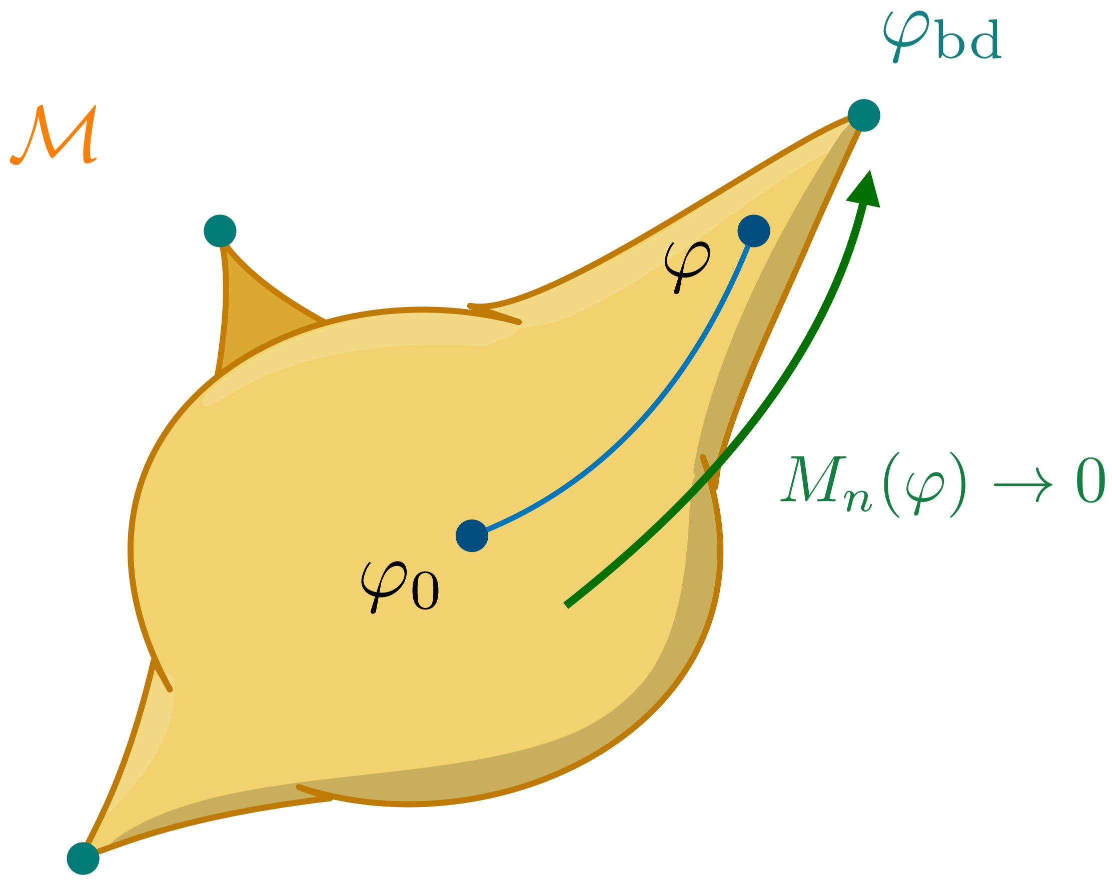
## SWAMPLAND DISTANCE CONJECTURE

**2.** Along paths leading to infinite field distance points, an infinite tower of states becomes exponentially light

$$M_n(\varphi) \sim M_n(\varphi_0) e^{-\lambda d(\varphi, \varphi_0)}$$

with $\lambda$ an *O(1)*-parameter.

**⇒ The EFT cutoff $\Lambda_{\text{EFT}}$ ought to be exponentially reduced.**



# ADDRESSING THE DISTANCE CONJECTURE

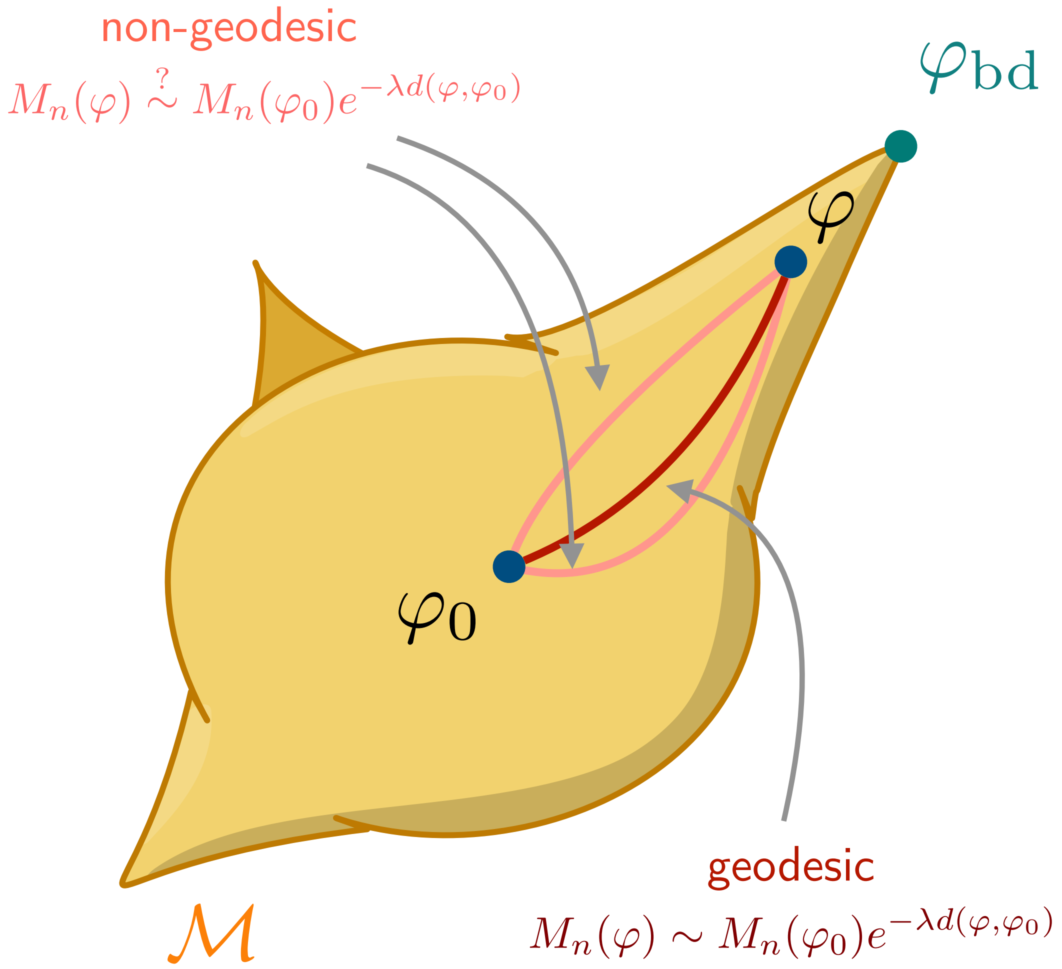
## I. PATH DEPENDENCE

Infinite distance points can be reached also via non-geodesic paths.

Consider an emergent tower along a geodesic path

$$M_n(\varphi) \sim M_n(\varphi_0) e^{-\lambda d(\varphi,\varphi_0)}$$

**Does this tower remain relevant also along other, non-geodesic paths?**



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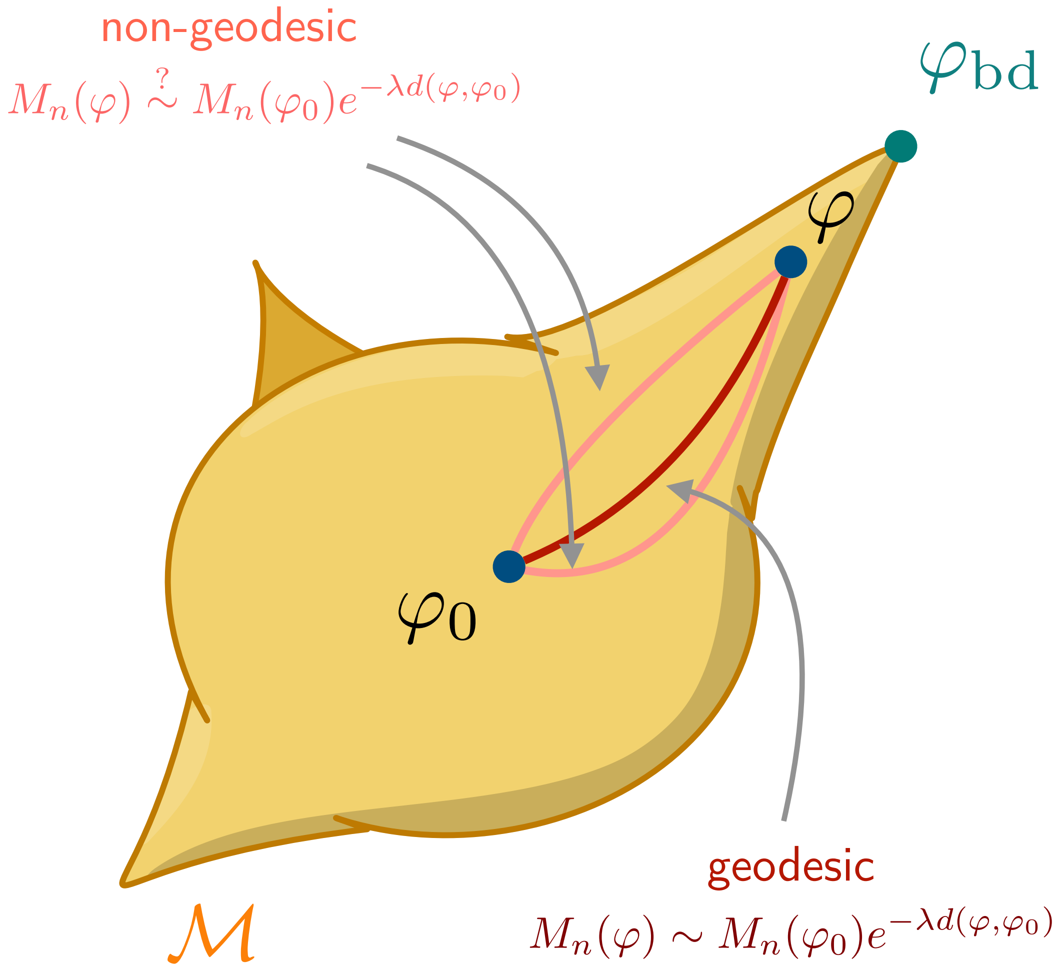

## II. COUNTING THE TOWERS OF STATES

How many towers are needed to realise the Distance Conjecture?

**Are they finite, or infinite in number?**

# THE TAMENESS CONJECTURE

[Grimm, 2021]

The **Tameness Conjecture** restricts the functional form of any EFT coupling, by assuming that they have to be definable in the $\mathbb{R}_{\text{an,exp}}$ *o-minimal structure*:

$$S^{(D)} = \int \Big( \frac{1}{2} M_{\text{P}}^{D-2} R * 1 - \frac{1}{2} M_{\text{P}}^{D-2} G_{ab}(\varphi, \lambda) \mathrm{d}\varphi^a \wedge * \mathrm{d}\varphi^b \\ - M_{\text{P}}^{D-2(p_{\mathcal{I}}+1)} f_{\mathcal{I}\mathcal{J}}(\varphi, \lambda) F^{\mathcal{I}}_{p_{\mathcal{I}}+1} \wedge * F^{\mathcal{J}}_{p_{\mathcal{J}}+1} - V(\varphi, \lambda) + \dots \Big)$$

Namely, **any** EFT coupling $g$ stems from the locus

$$\exists\, x_1, \dots, x_l : \qquad \begin{aligned} &P_i(\varphi, \lambda, x, g, f_1, \dots, f_m, e^{\varphi}, e^{\lambda}, e^{x}, e^{g}) = 0\,, \\ &Q_j(\varphi, \lambda, x, g, f_1, \dots, f_m, e^{\varphi}, e^{\lambda}, e^{x}, e^{g}) > 0\,, \end{aligned}$$

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moduli

parameters
(e.g. fluxes)

auxiliary
variables

restricted analytic
functions

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Tame couplings

$$g(\varphi) = e^{\alpha\varphi} \quad \text{or} \quad g(\varphi) = \sum_i \alpha_i \varphi^{\beta_i}$$

Non-tame couplings

$$g(\varphi) = \sin\varphi$$

# SPECIAL CLASSES OF TAME COUPLINGS

[Bakker, Klinger, Tsimerman, 2020]

Consider a tame coupling

$$g(\phi, s)$$

$\phi^\alpha$ Compact fields: $\phi^\alpha \in [0, 1]$

$s^i$ Non-compact fields,
with boundary located at $s^i \to \infty$

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We focus on two classes of tame couplings:

**Monomially tamed couplings, with definite growth properties**

These are couplings of the general form

$$g(\phi, s) = \sum_{\mathbf{m}} \rho_{\mathbf{m}}(e^{-s^i}, \phi^\alpha)(s^1)^{m_1} \cdots (s^n)^{m_n}$$

restricted analytic functions

which can be well-approximated by monomials

$$g(\phi, s) \sim (s^1)^{k_1} \cdots (s^n)^{k_n} \qquad \text{on} \quad \mathcal{U}$$

or, there exist $C_1, C_2 > 0$:

$$C_1 (s^1)^{k_1} \cdots (s^n)^{k_n} < g(\phi, s) < C_2 (s^1)^{k_1} \cdots (s^n)^{k_n} \qquad \text{on} \quad \mathcal{U}$$

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restricted analytic functions

which are, **at most**, upper bounded by a monomial

$$g(\phi, s) \prec (s^1)^{k_1} \cdots (s^n)^{k_n} \qquad \text{on} \quad \mathcal{U}$$

or, there exists $C > 0$:

$$g(\phi, s) < C(s^1)^{k_1} \cdots (s^n)^{k_n} \qquad \text{on} \quad \mathcal{U}$$

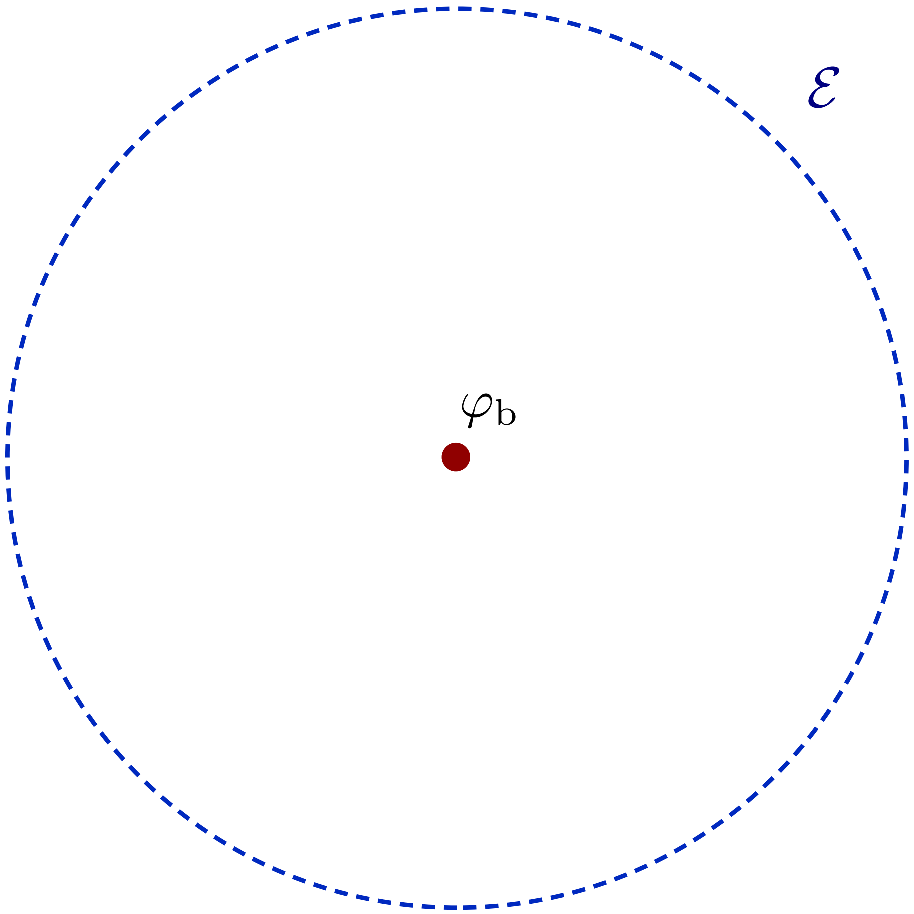
# THE DISTANCE CONJECTURE AND PATH-INDEPENDENCE

Assumptions

$M_n^{(a)}$ $\quad e^{-\lambda d(s)}$ polynomially tamed

*i.e.* $\quad M_n^{(a)} \prec (s^1)^{p_1} \cdots (s^n)^{p_n} \qquad e^{-\lambda d(s)} \prec (s^1)^{m_1} \cdots (s^n)^{m_n}$

$\Rightarrow$ *In general, no leading term can be singled out in the near boundary region.*



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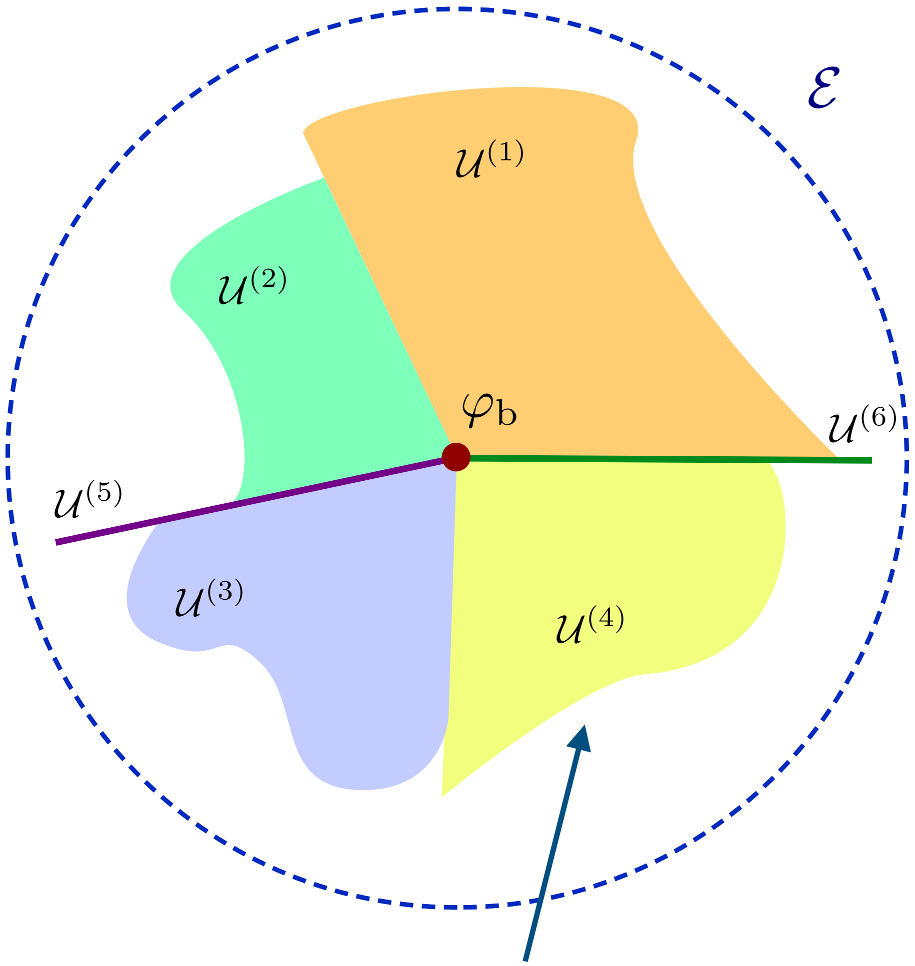
⇒ *In general, no leading term can be singled out in the near boundary region.*

We **partition** the near-boundary region in subsets $\mathscr{U}^{(A)}$ obeying two properties:

- On each $\mathscr{U}^{(A)}$, $M_n^{(a)}(s)$ and $e^{-\lambda d(s)}$ are strictly decreasing (*Monotonicity Theorem*);
- On each $\mathscr{U}^{(A)}$, $M_n^{(a)}(s)$ and $e^{-\lambda d(s)}$ display a definite leading behavior. *For instance, they can both be monomially tamed*



$M_n^{(a)} \sim (s^1)^{p_1} \cdots (s^n)^{p_n} \sim e^{-\lambda d}$

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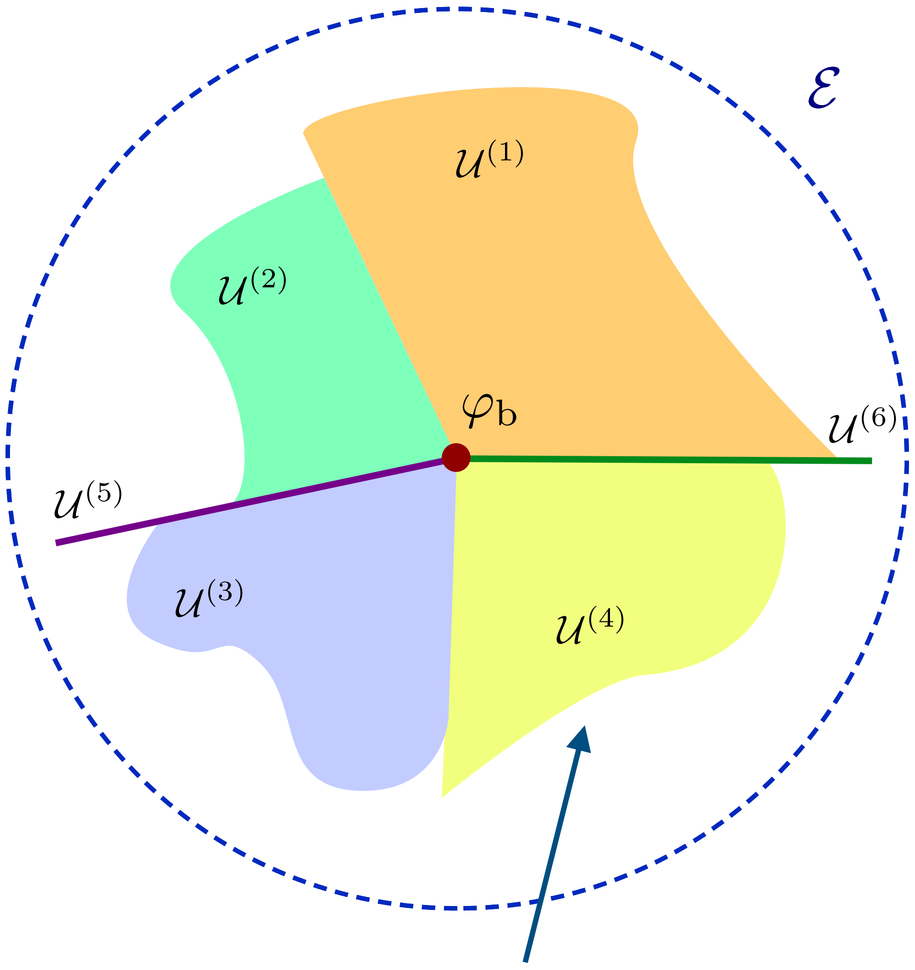

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**Path-indipendency**: If on each $\mathscr{U}^{(A)}$ we can establish

$$M_n^{(a)}(s) \sim e^{-\lambda d(s)} \sim f_{\rm leading}(s)$$

the Distance Conjecture is realized along **every** path in $\mathscr{U}^{(A)}$.

# THE DISTANCE CONJECTURE AND FINITENESS

## Assumptions

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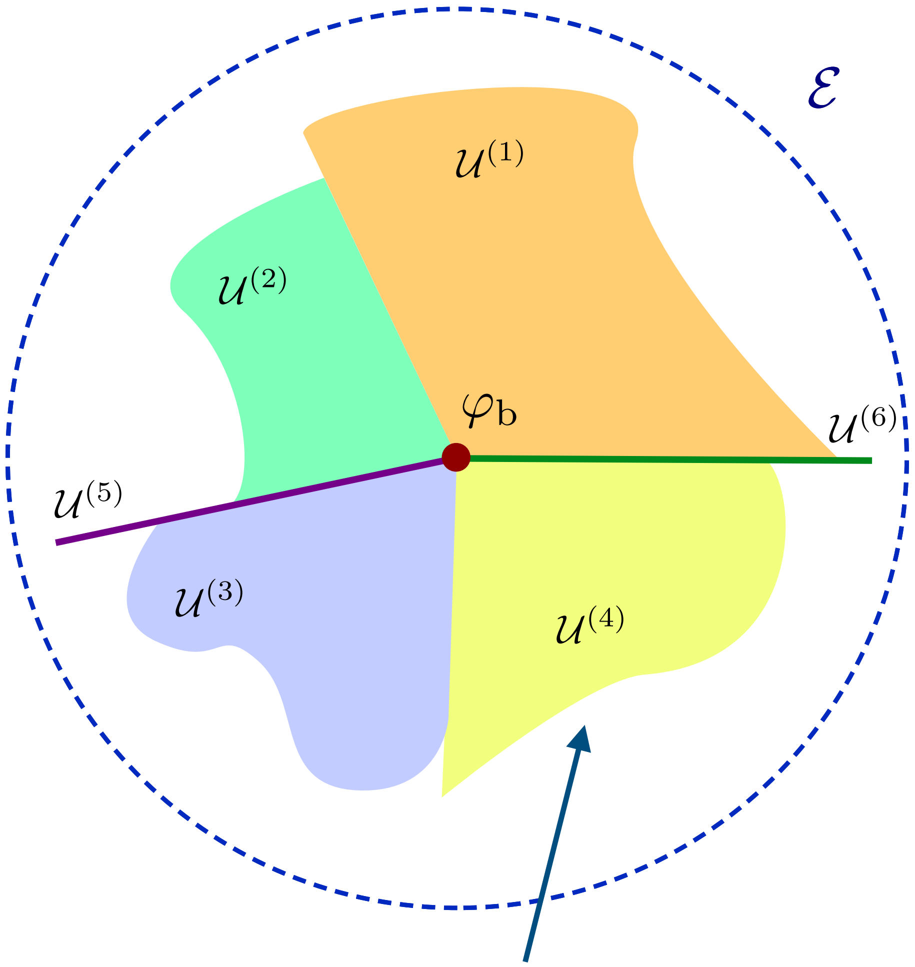

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**Finiteness of the number of the infinite towers of states**:

Only a **finite** number of sets $\mathscr{U}^{(A)}$ is required to realize

$$M_n^{(a)}(s) \sim e^{-\lambda d(s)} \sim f_{\rm leading}(s)$$

path-independently.

⇒ Only a **finite** number of tower of states is required to realize the Distance Conjecture.

# HOW TO PROBE TAME COUPLINGS

Polynomially tame couplings can be tested via the **Curve reduction theorem**.
For the Distance Conjecture, this implies the following:

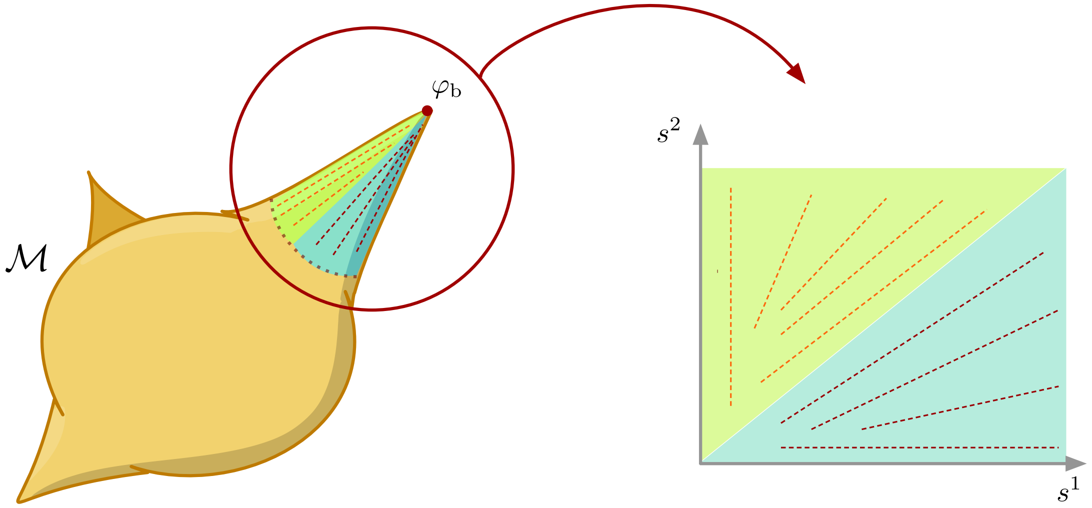
- Decompose the near-boundary regions $\mathscr{U}^{(A)}$ in sets such that:

$$M_n(s) \qquad e^{-\lambda d(s)} \qquad \text{monomially tamed}$$

- If $M_n \sim e^{-\lambda d}$ on the linear paths

$$s^i = s_0^i + e^i\sigma\,, \qquad \phi^i = \text{const.}$$

then, the Distance Conjecture holds path-independently on $\mathscr{U}^{(A)}$.



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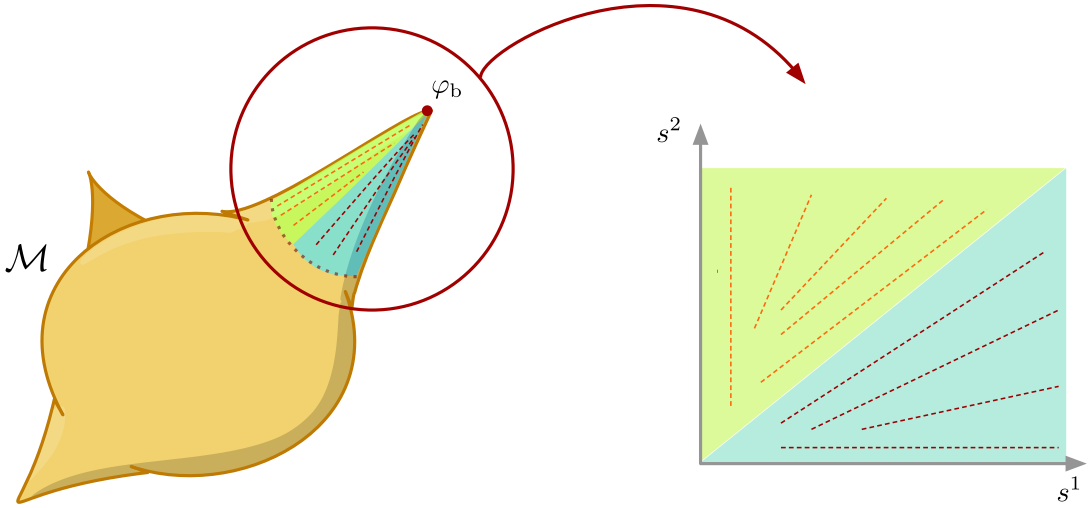

In 4D, the paths

$$s^i = s_0^i + e^i \sigma \,, \qquad \phi^i = \text{const.}$$

in field space can be regarded as backreaction of cosmic strings.
⇒ **Cosmic strings strings are good candidates to probe the near-boundary physics.**
Reminiscent of the **Distant Axionic String Conjecture**. [SL, Marchesano, Martucci, Valenzuela, '21]

# CONCLUSIONS AND FUTURE OUTLOOK

Assuming that the couplings are “tame” allows for better addressing questions about the EFT structures in generality.

We have showed:

- how the **Distance Conjecture** can be realized **path-independently**;
- that only a **finite** number of towers is needed in order to realize the Distance Conjecture;
- how to **probe** some special classes of tame couplings via **cosmic strings**.

Some open questions:

- Are polynomially and monomially tamed functions enough to examine **all** the corners of EFT moduli spaces?
- Can we address other **Swampland questions** via Tameness?
  [works in progress with T.Grimm, M. Van Vliet, T. Van Vuren]
- Can Tame structure be employed to test **non-supersymmetric settings**?

Thanks for

your attention!

# BACKUP SLIDES

# AN EXAMPLE: F-THEORY / TYPE IIB EFTS

Consider 4D EFTs obtained compactifying Type IIB string theory over a Calabi-Yau three-fold. The couplings of the vector multiplet sector are fully determined by the Calabi-Yau periods

$$\Pi^{\mathcal{I}}(\varphi) = \int_{\Gamma_{\mathcal{I}}} \Omega$$

$\varphi^i = a^i + \mathbf{i} s^i$:$h^{2,1}$-complex structure moduli

**EFT couplings, defined on $\Sigma = \{s^1 > s^2 > \ldots > s^n > 1\}$:**

Kähler potential

$$e^{-K^{\mathrm{cs}}} = \mathbf{i} \int_Y \Omega \wedge \bar{\Omega} = \mathbf{i}\, \mathbf{\Pi}^T \eta \bar{\mathbf{\Pi}}$$

Kähler metric

$$K^{\mathrm{cs}}_{i\bar{\jmath}} = \frac{\partial^2 K^{\mathrm{cs}}}{\partial \varphi^i \partial \bar{\varphi}^{\bar{\jmath}}}$$

Masses of D3-particles

$$M_{\mathbf{q}} = |\mathcal{Z}_{\mathbf{q}}| = e^{\frac{K^{\mathrm{cs}}}{2}} \left| \int_Y q \wedge \Omega \right|$$

Charges of D3-particles/ gauge couplings

$$\mathcal{Q}^2_{\mathbf{q}} = \frac{1}{2} \int_Y q \wedge \star q$$

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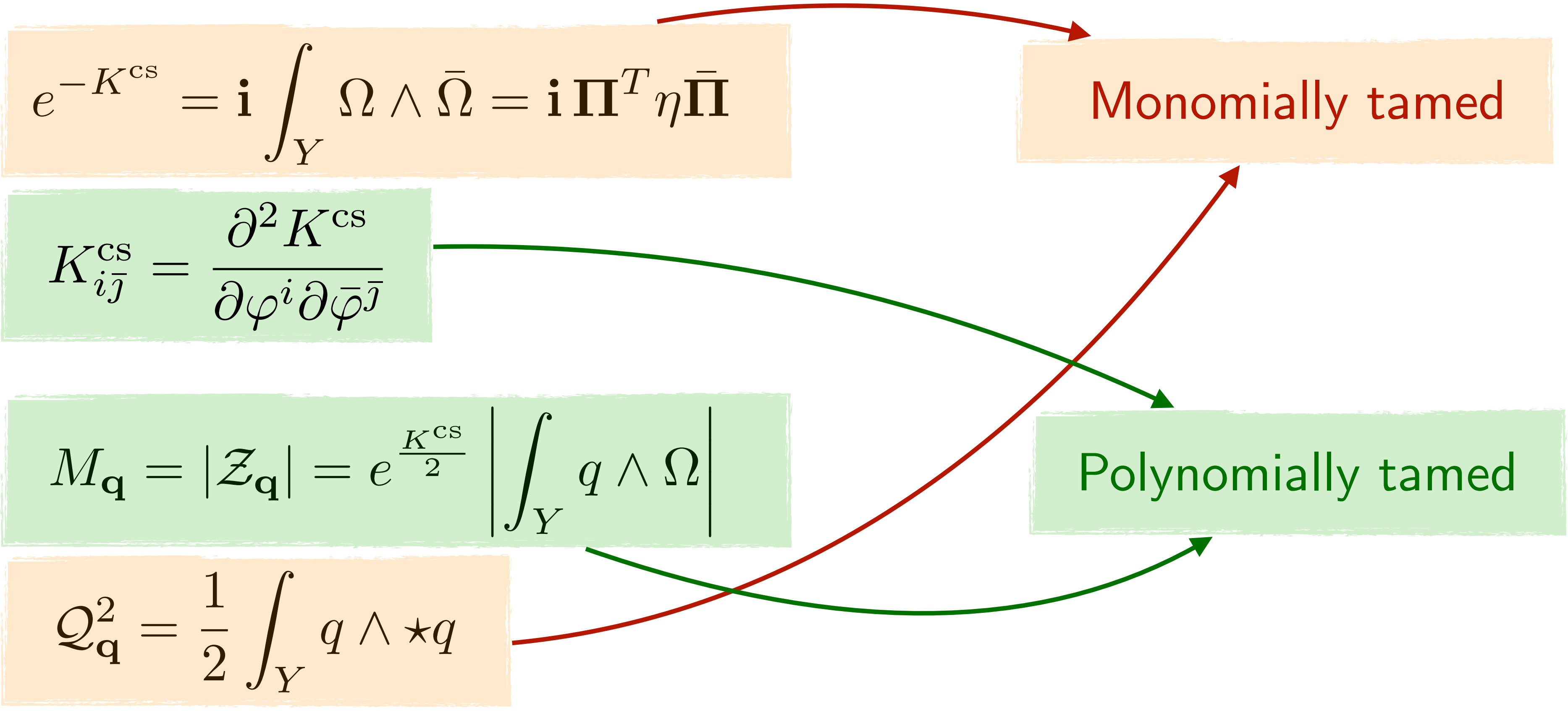

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